

## Weighted hardy inequalities with sharp constants

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### Abstract

Using the Bessel functions we obtain several weighted Hardy inequalities with sharp constants. The following inequality for absolutely continuous functions is a simple example: If  $p$  and  $\nu$  are positive numbers, and  $f: [0, 1] \rightarrow \mathbb{R}$  satisfies the conditions  $f(0) = 0$  and  $x^{1/2-p/2}f' \in L^2(0, 1)$ , then, where  $F_\nu(x) = \sqrt{x}J_\nu(j_{\nu-1}x/(2\nu))$ ,  $J_\nu$  is the Bessel function of order  $\nu$  and  $j_{\nu-1}$  is the first positive zero of  $J_{\nu-1}$ . In the general case we have to introduce constants  $z = \lambda_\nu(2/q)$  as the first positive root of the Lamb equation  $J_\nu(z) + qzJ'_\nu(z) = 0$  and the functions  $z = z(q)$  that may be found as the solution of the initial values problem. © 2010 Pleiades Publishing, Ltd.

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### Keywords

Bessel function, Hardy inequality, Lamb constant